TRINITY COLLEGE FOR WOMEN NAMAKKAL
Department of Mathematics

ADVANCED ALGEBRA
23PMA04– **EVEN SEMESTER**

GALOIS THEORY

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FIELDS AND AUTOMORPHISMS

A field F is a set with invertible multiplication and addition such that the distributive property holds. It is a generalization of Q.

A field homomorphism is a map to another field that commutes with both operations.

They are either injective or trivial.

A field automorphism is an isomorphism from F to itself.

Field automorphism are invertible and can be composed, so they can form a group.

PRIME SUBFIELDS:

The field F has a multiplicative

identity 1. Let σ be any automorphism of F.

Let k be an element of the form $(1 + \cdots +$

If F is finite, it is the ring Z/pZ for some prime p.

FUNDAMENTAL IDEAS

An object's group of symmetries contains important information about that object.

Polynomials define algebraic behavior and thus can be used to create new mathematical objects.

Galois theory is usually described as the study of field automorphisms and polynomials over fields.

FIELD EXTENSIONS:

When F is a subfield of E, we say that E is an extension of F and write E and E/F. Extensions can be created by adjoining roots of polynomials.

The automorphism group of a field is Aut(E). The automorphism group of a field E fixing a subfield F is Aut(E/F). For a prime subfield Q, Aut(E)=Aut(E/Q).

MORE ON FIELD EXTENSIONS:

Example:

Adjoining $\sqrt{2}$ to Q to get Q$(\sqrt{2})$

$\sqrt{2}$ is a root of the irreducible polynomial $x^2 - 2$; we define $Q(\sqrt{2})$ as the set of elements of the form $a + b\sqrt{2}$ where a and b are rational. This is isomorphic to the quotient $Q[x]/(x^2 - 2)$.

Another example:

$x^2 + 1$ is irreducible over R. We define i as one of its roots and adjoin it to get

$\sqrt{2}$ is a root of the irreducible polynomial $x^2 - 2$; we define $Q(\sqrt{2})$ as the set of elements of the form $a + b\sqrt{2}$ where a and b are rational.

This is isomorphic to the quotient $Q[x]/(x^2 - 2)$.

THANK YOU

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