TRINITY COLLEGE FOR WOMEN NAMAKKAL
Department of Mathematics

ALGEBRA AND TRIGONOMETRY
23UMA01 – Odd Semester

Introduction to Hyperbolic Functions

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HYPERBOLIC FUNCTIONS

Certain combinations of the exponential functions e^x and e^{-x} arise so frequently in mathematics and its applications that they are given special names.

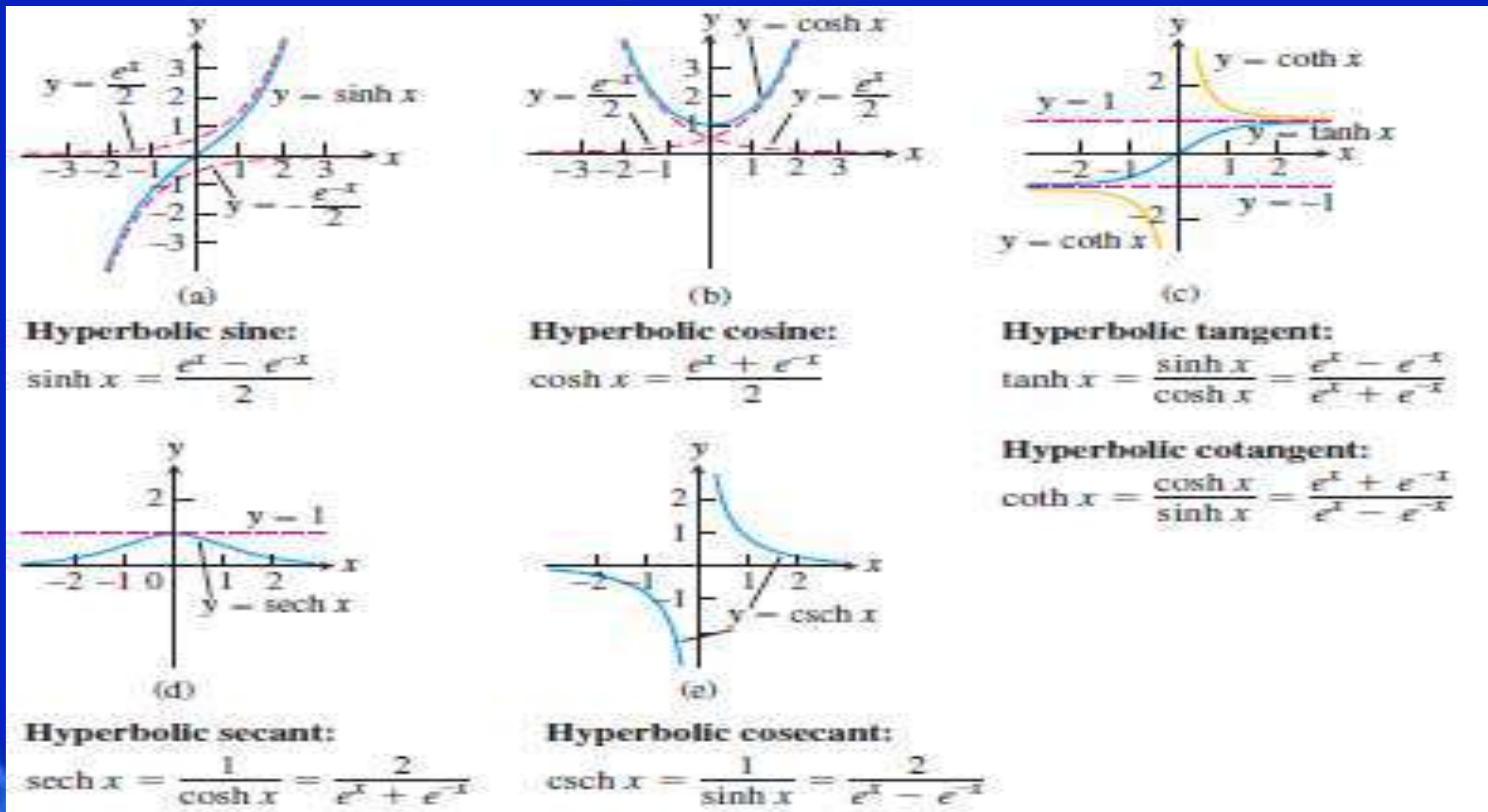
$$y = \frac{e^x - e^{-x}}{2} \qquad y = \frac{e^x + e^{-x}}{2}$$

The behavior of these functions shows such remarkable paralles to trig functions, that they have been given similar names.

Hyperbolic sine: sin h(x)= $\frac{e^x - e^{-x}}{2}$

Hyperbolic Cosine: cosh(x) = $\frac{e^x + e^{-x}}{2}$

The six basis Hyperbolic Functions



Prove that cosh(x − y) = cosh x cosh y − sinh x sinh y

Solution

$$\cosh x \cosh y = \frac{1}{2}(e^{x} + e^{-x}) \times \frac{1}{2}(e^{y} + e^{-y})$$

$$= \frac{1}{4}(e^{x+y} + e^{x-y} + e^{-(x-y)} + e^{-(x+y)})$$

$$\sinh x \sinh y = \frac{1}{2}(e^{x} - e^{-x}) \times \frac{1}{2}(e^{y} - e^{-y})$$

$$= \frac{1}{4}(e^{x+y} - e^{x-y} - e^{-(x-y)} + e^{-(x+y)})$$

Example: *Find* $\frac{d}{dx}(\sinh u)$

$$\frac{d}{dx}(\sinh u) = \frac{d}{dx}\left(\frac{e^{u} - e^{-u}}{2}\right) = \frac{e^{u}\frac{du}{dx} + e^{-u}\frac{du}{dx}}{2} = \frac{du}{dx}\left(\frac{e^{u} + e^{-u}}{2}\right)$$

$$= \cosh u \frac{du}{dx}$$

Example: *Find* $\frac{d}{dx}(\operatorname{csch} u)$

$$= \frac{d}{dx}\left(\frac{1}{\sinh u}\right) = \frac{-\cosh u \frac{du}{dx}}{sinh^{2}u} = -\coth u \operatorname{csch} u \frac{du}{dx}$$

Example: *Find* $\lim_{x\to\infty}(cosh^{-1}x - \ln x)$

$$y = cosh^{-1}x \quad \rightarrow \quad x = \cosh y = \frac{e^y + e^{-y}}{2}$$

$$e^y + e^{-y} = 2x$$

$$e^{2y} + 1 = 2e^y x$$

$$e^{2y} - 2xe^y + 1 = 0$$

$$e^y = \frac{2x \mp \sqrt{4x^2 - 4}}{2} = x \mp \sqrt{x^2 - 1}$$

$$e^y = x + \sqrt{x^2 - 1}$$

$$y = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$cosh^{-1}x = ln\left(x + \sqrt{x^2 - 1}\right)$$

$$= \lim_{x\to\infty}\left(\ln\left(x + \sqrt{x^2 - 1}\right) - \ln x\right)$$

$$= \lim_{x\to\infty}\left(\ln\frac{x + \sqrt{x^2 - 1}}{x}\right)$$

$$= \lim_{x\to\infty}\left(\ln\frac{1 + \sqrt{1 - \frac{1}{x^2}}}{1}\right) = \ln 2$$

Application of Hyperbolic functions

Hyperbolic functions show up in many real-life situations. For example, they are related to the curve one traces out when chasing an object that is moving linearly. They also define the shape of a chain being held by its endpoints and are used to design arches that will provide stability to structures.

In the field of engineering,

Electrical lines suspended freely between two poles, or any idealized hanging chain or cable supported just at its ends and hanging under its own weight can be described using hyperbolic functions.

THANK YOU

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