TRINITY COLLEGE FOR WOMEN NAMAKKAL

Department of Physics

Mathematical Physics

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Laplace transforms Definition

$$F(s) = L\{f(t)\} = \int_{0}^{\infty} f(t)e^{-st}dt$$

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds$$

More Definitions

$$f(t) = 1$$

$$F(s) = \int_0^{\infty} e^{-st} dt = -\frac{1}{s}(0 - 1) = \frac{1}{s}$$

$$f(t) = e^{-at}$$

$$F(s) = \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt = \frac{1}{s + a}$$

$$f(t) = \sin\ t$$

$$F(s) = \int_0^{\infty} e^{-st} \sin(\ t) dt$$

Evaluating F(s)=L{f(t)}

$$u = e^{-st}, du = -se^{-st}dt$$
$$dv = \sin(t)dt, v = -\cos(t)$$

$$\therefore \int_0^\infty e^{-st}\sin(t)dt = [-e^{-st}\cos(t)]_0^\infty - s\int_0^\infty e^{-st}\cos(t)dt =$$

$$-e^{-st}(1) - s\int_0^\infty e^{-st}\cos(t)dt$$

$$u = e^{-st}, du = -se^{-st}dt$$
$$dv = \cos(t)dt, v = \sin(t)$$

$$\therefore \int_0^\infty e^{-st}\cos(t)dt =$$

$$[-e^{-st}\sin(t)]_0^\infty + s\int_0^\infty e^{-st}\sin(t)dt = -e^{-st}(0) + s\int_0^\infty e^{-st}\sin(t)dt$$

$$\int_0^\infty se^{-st}\sin(t)dt = 1 - s^2\int_0^\infty e^{-st}\sin(t)dt =$$

$$(1+s^2)\int_0^\infty e^{-st}\sin(t)dt = 1$$

$$\int_0^\infty e^{-st}\sin(t)dt = \frac{1}{1+s^2}$$

Examples

$$f(t) = u(t) \Leftrightarrow F(s) = \frac{1}{s}$$

$$f(t) = e^{-at}u(t) \Leftrightarrow F(s) = \frac{1}{s+a}$$

$$f(t) = \cos(t)u(t) \Leftrightarrow F(s) = \frac{s}{s^2+1}$$

$$f(t) = \sin(t)u(t) \Leftrightarrow F(s) = \frac{1}{s^2+1}$$

Step functions in Laplace Transform

- Unit step function definition:

$$u(t) = 1,\ t \geq 0$$

$$u(t) = 0,\ t < 0$$

- Used in conjunction with $f(t) \rightarrow f(t)u(t)$ because of Laplace integral limits:

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$$

Properties of Laplace Transforms

- Linearity
- Scaling in time
- Time shift
- “frequency” or s-plane shift

1. Scaling in Time

$$L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$$

2. Linearity

$$L\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 F_1(s) + c_2 F_2(s)$$

3. S-plane (frequency) shift

$$L\{e^{-at}f(t)\} = F(s+a)$$

4. Time Shift

$$L\{f(t-t_0)u(t-t_0)\} = e^{-st_0}F(s)$$

THANK YOU

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