TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

GRAPH THEORY & APPLICATIONS
23PMAE02 - EVEN Semester

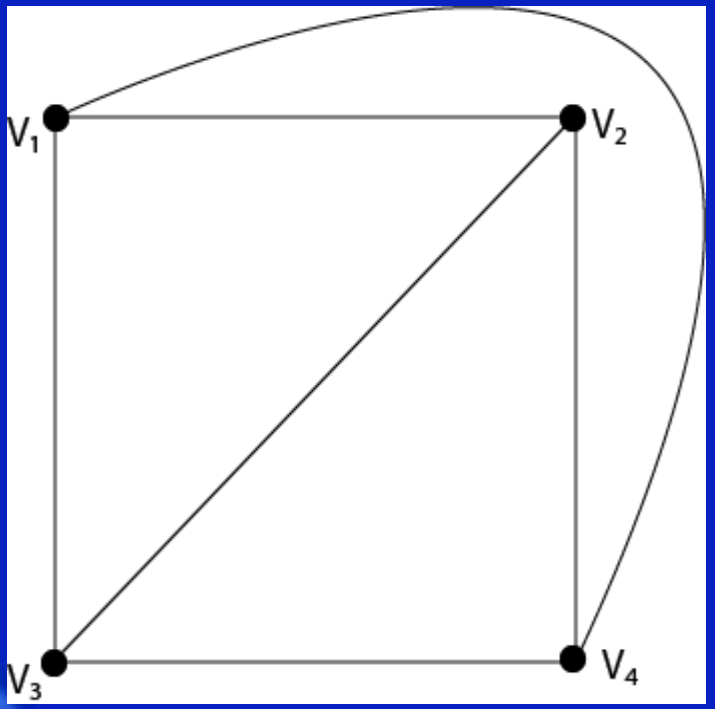
Introduction to Planar & Nonplanar Graphs

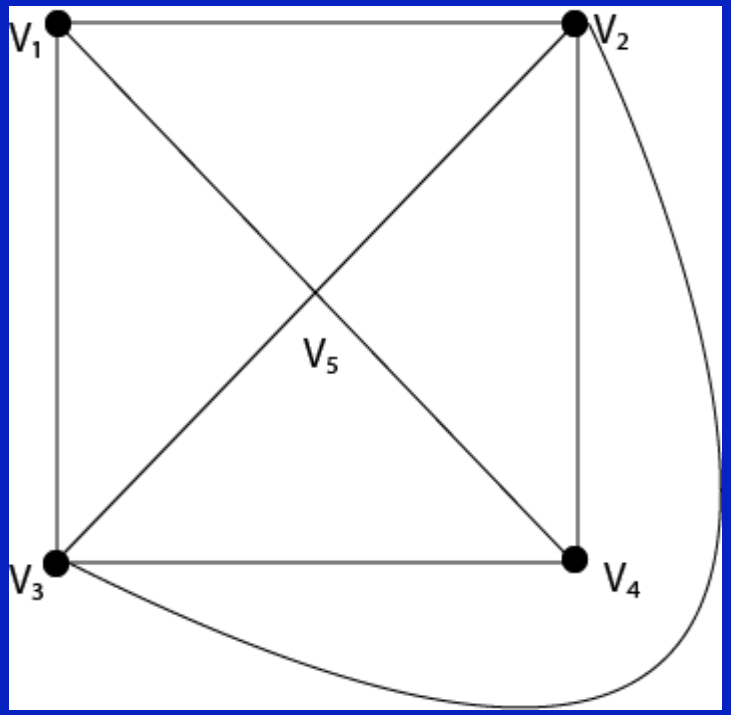
Presented by
Mrs. S. Madhukaraveni
Assistant Professor
Department of Mathematics
http://www.trinitycollegenkl.edu.in/

A graph is said to be planar if it can be drawn in a plane so that no edge cross.

Example:

The graph shown in fig is planar graph.





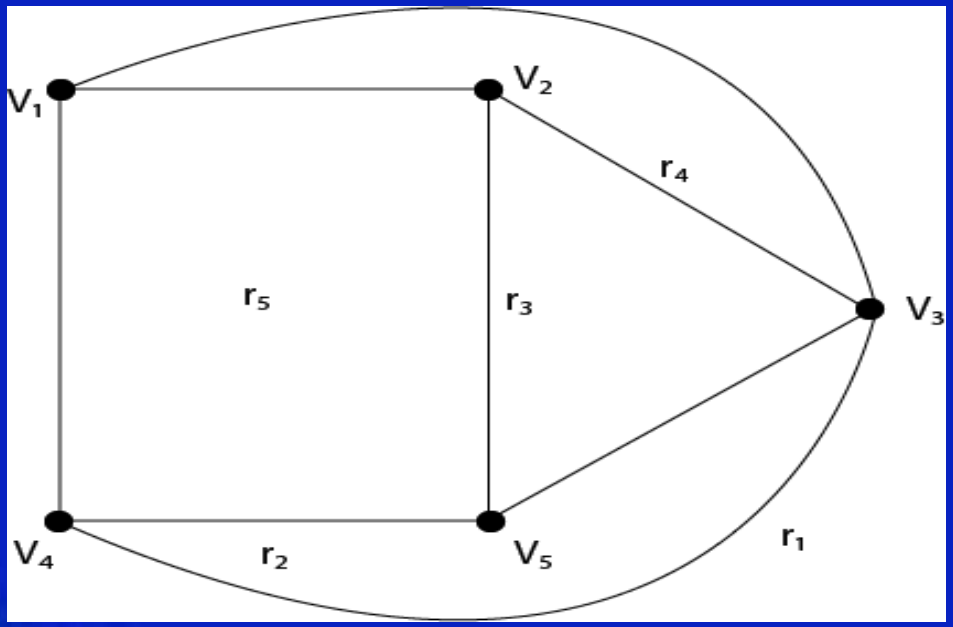
Region of a Graph

Consider a planar graph G=(V,E).A region is defined to be an area of the plane that is bounded by edges and cannot be further subdivided. A planar graph divides the plans into one or more regions. One of these regions will be infinite.

Finite Region

If the area of the region is finite, then that region is called a finite region.

Infinite Region: If the area of the region is infinite, that region is called a infinite region. A planar graph has only one infinite region.
Example: Consider the graph shown in Fig. Determine the number of regions, finite regions and an infinite region.



Solution:

There are five regions in the above graph, i.e. r_1, r_2, r_3, r_4, r_5. There are four finite regions in the graph, i.e., r_2, r_3, r_4, r_5. There is only one finite region, i.e., r_1

Properties of Planar Graphs:

1. If a connected planar graph G has e edges and r regions, then $r \leq e$.
2. If a connected planar graph G has e edges, v vertices, and r regions, then $v-e+r=2$.
3. If a connected planar graph G has e edges and v vertices, then $3v-e \geq 6$.

4. A complete graph K_n is a planar if and only if $n<5$.

5.A complete bipartite graph K_{mn} is planar if and only if $m<3$ or $n>3$.

Example:

Prove that complete graph K_4 is planar.

Solution:

The complete graph K_4 contains 4 vertices and 6 edges.

We know that for a connected planar graph $3v-e\geq 6$.Hence for K_4, we have $3x4-6=6$ which satisfies the property (3).

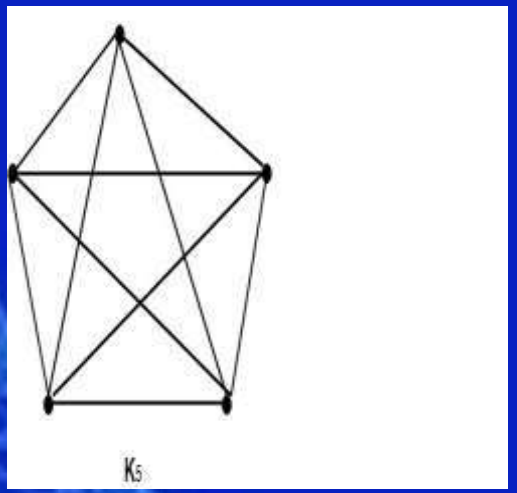
Thus K_4 is a planar graph. Hence Proved.

Non-Planar Graph:

A graph is said to be non planar if it cannot be drawn in a plane so that no edge cross.

Example:

The graphs shown in fig are non planar graphs.



These graphs cannot be drawn in a plane so that no edges cross hence they are non-planar graphs.

Properties of Non-Planar Graphs:
A graph is non-planar if and only if it contains a subgraph homeomorphic to K_5 or $K_{3,3}$

Example1:
Show that K_5 is non-planar.
Solution:
The complete graph K_5 contains 5 vertices and 10 edges.
Now, for a connected planar graph $3v-e \geq 6$.
Hence, for K_5, we have $3 \times 5-10=5$ (which does not satisfy property 3 because it must be greater than or equal to 6).
Thus, K_5 is a non-planar graph.

Example2: Show that the graphs shown in fig are non-planar by finding a subgraph homeomorphic to K_5 or $K_{3,3}$.

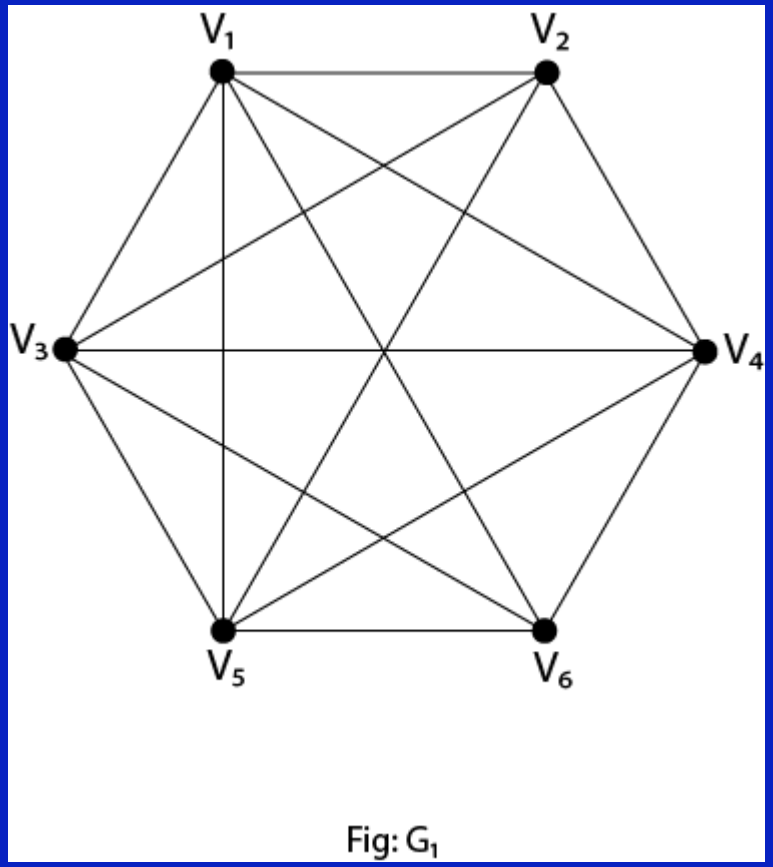


Fig: G_1

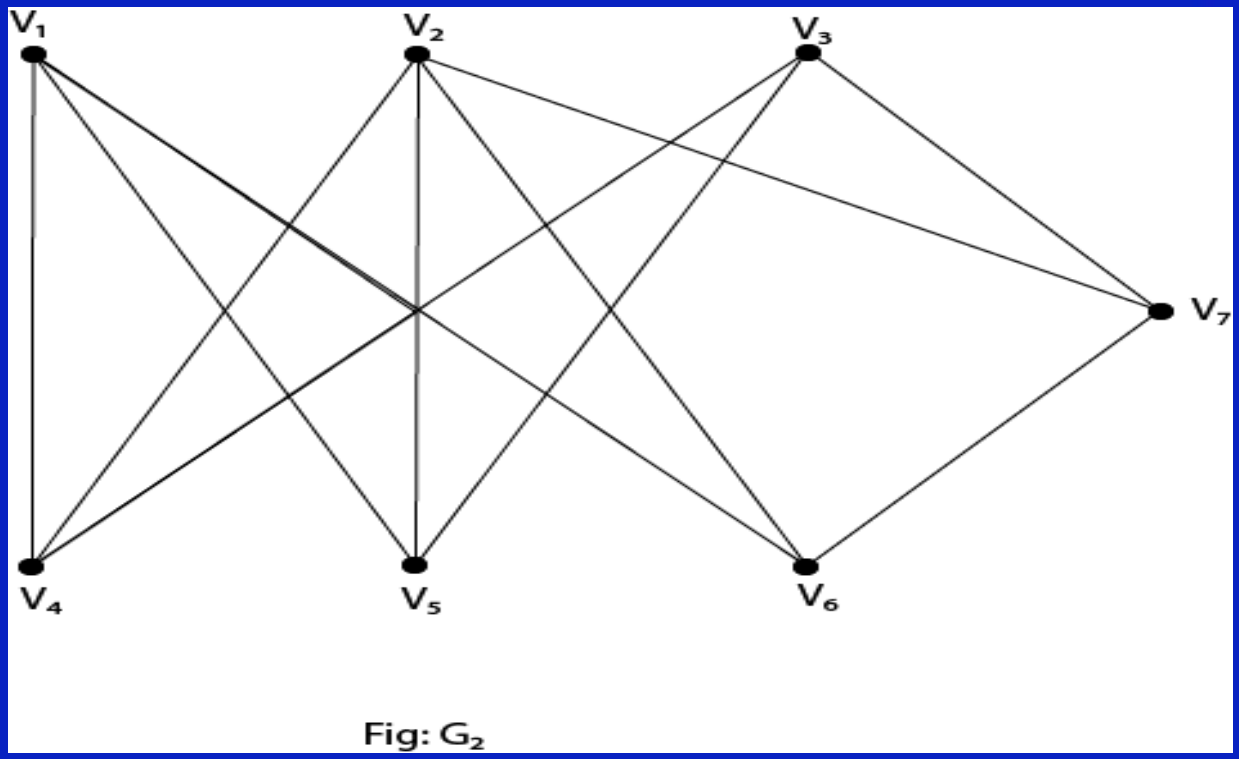


Fig: G_2

Solution: If we remove the edges (V_1,V_4),$(V_3,V_4)$and (V_5,V_4) the graph G_1,becomes homeomorphic to K_5.Hence it is non-planar.
If we remove the edge $V_{2,V}7)$ the graph G_2 becomes homeomorphic to $K_{3,3}$.Hence it is a non-planar.

THANK YOU

http://www.trinitycollegenkl.edu.in/