TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

INTEGRAL CALCULUS

23UMA04-Even Semester

Topic: Double Integral and Volume

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Introduction of Double Integral

Double Integral over a region R may be evaluated by two successive integrals. In this section in the first part we see how to express a double integral as an iterated integral, which can then be evaluated by calculating two single integrals over the rectangle.

Uses of Double Integral

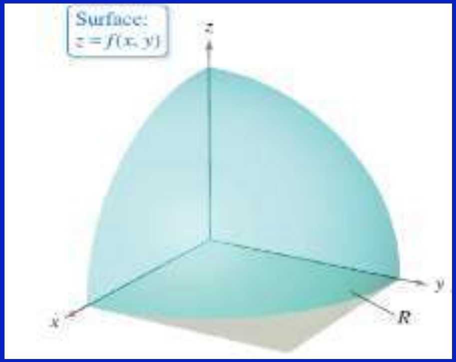
- Use a double integral to represent the volume of a solid region and use properties of double integrals.
- Evaluate a double integral as an iterated integral.
- Find the average value of a function over a region.

Double Integrals and Volume of a Solid Region

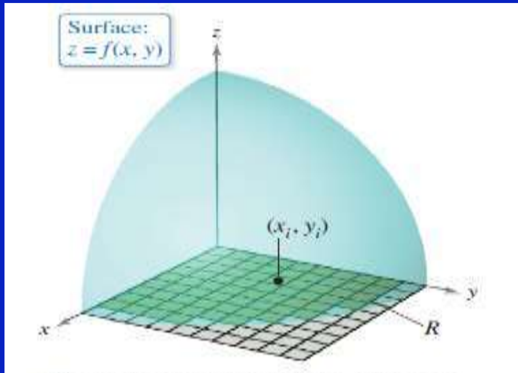
Consider a continuous function f such that $f(x, y) \geq 0$ for all (x, y) in a region R in the xy-plane. The goal is to find the volume of the solid region lying between the surface given by $z = f(x, y)$

Surface lying above the *xy*-plane

and the *xy*-plane, as shown in Figure.



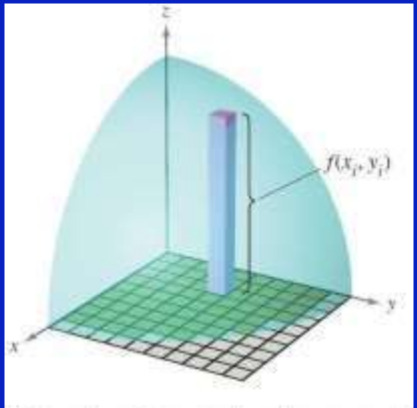
You can begin by superimposing a rectangular grid over the region, as shown in Figure



The rectangles lying within R form an inner partition of R.

The rectangles lying entirely within *R* form an inner partition Δ, whose **norm** ||Δ|| is defined as the length of the longest diagonal of the *n* rectangles.

Next, choose a point (x_i, y_i) in each rectangle and form the rectangular prism whose height is $f(x_i, y_i)$, as shown in Figure



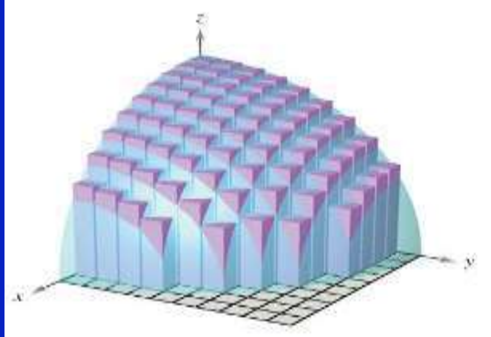
Rectangular prism whose base has an area of ΔA_i and whose height is $f(x_i, y_i)$

Because the area of the *i*th rectangle is ΔA_i Area of *i*th rectangle

it follows that the volume of the *i*th prism is $f(x_i, y_i)\,\Delta A_i$ Volume of *i*th prism and you can approximate the volume of the solid region by the Riemann sum of the volumes of all *n* prisms,

$$\sum_{i=1}^{n} f(x_i, y_i)\,\Delta A_i \qquad \text{Riemann sum}$$

as shown in Figure

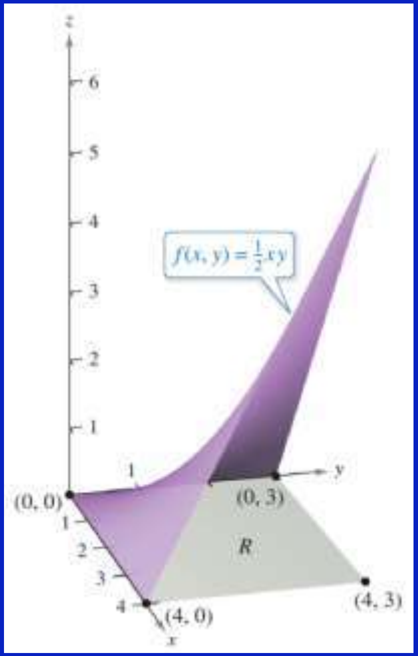


Volume approximated by rectangular prisms

This approximation can be improved by tightening the mesh of the grid to form smaller and smaller rectangles, Example 1:

Finding the Average Value of a Function

Find the average value of $f(x, y) = \frac{1}{2}xy$ over the region *R*, where *R* is a rectangle with vertices (0, 0), (4, 0), (4, 3), and (0, 3).



Solution:

The area of the rectangular region *R* is $A = 12$

The average value is given by

$$\frac{1}{A}\int_R\int f(x, y)\, dA = \frac{1}{12}\int_0^4\int_0^3 \frac{1}{2}xy\, dy\, dx$$

$$= \frac{1}{12}\int_0^4 \frac{1}{4}xy^2\Big]_0^3 dx$$

$$= \left(\frac{1}{12}\right)\left(\frac{9}{4}\right)\int_0^4 x\, dx$$

$$= \frac{3}{16}\left[\frac{1}{2}x^2\right]_0^4$$

$$= \left(\frac{3}{16}\right)(8)$$

$$= \frac{3}{2}.$$

THANK YOU

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