TRINITY COLLEGE FOR WOMEN
NAMAKKAL
Department of Mathematics

REAL ANALYSIS- II
23PMA06- EVEN SEMESTER

INTRODUCTION TO MEASURE THEORY

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LEBESGUE OUTER MEASURE

The Lebesgue outer measure (or) outer measure of a set is given by

$$m^*(\text{A}) = \text{Inf} \sum l(I_n)$$

Where the infimum is taken over all finite or countable collections of intervals I_n such that A$\subseteq \bigcup I_n$.

PROPERTIES:

- ⋈ $m^*(A) \geq 0$.
- ⋈ $m^*(\varphi) = 0$
- ⋈ $m^*(A) \leq m^*(B)$ if $A \subseteq B$

EXAMPLE:

For any set A $m^*(A) = m^*(A+x)$, where A+x=[y+x;y∈A].

That means outer measure is translation invarient.

RESULT:

The outer measure of an interval equals its length.

LEBESGUE MEASURABLE (OR) MEASURABLE

The set E is lebesgue measureble or measurable if for each set A we have

$$m^*(\mathrm{A}) \geq m^*(\mathrm{A} \cap \mathrm{E}) + m^*(\mathrm{A} \cap c\,E)$$

Here c E represents the complement of E.

EXAMPLE:

If $m^*(A) = 0$ then E is measurable.

PROPERTY IF COUNTABLE SUBADDITIVITY:

For any sequence of set $\{F_i\}$,

$$m^*(\cup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} m^*(E_i)$$

σ - ALEGBRA

A class of subsets of an arbitrary space X is said to be $\sigma -$ Algebra if X belongs to the class and the class is closed under the formation of countable unions and of complements.

We will denote by $\mathcal{M}$ the class of lebesgue measurable sets.

RESULTS:

- The class $\mathcal{M}$ is a $\sigma -$ Algebra.

- If F $\in \mathcal{M}$ and $m^*(\mathrm{F}\Delta G) = 0$, Then G is measurable.

- Every interval is measurable.

- If $\{E_i\}$ is any sequence of disjoint measurable sets, then

$$m^*(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} m^*(E_i)$$

BOREL SETS

We denote by $\mathfrak{B}$ the $\sigma -$ Algebra generated by the class of intervals of the form [a,b) its members are called the Borel sets of R.

* $\mathfrak{B} \subseteq \mathcal{M}$, That is every Borel sets is measurable.

EXAMPLES:

* For any set A there exists a measurable set E containing A and such that m^*(A) = m(E).

* Every non-empty open sets has positive measure.

* There exist uncountable sets of zero measure.

* Every countable set has measure zero.

THANK YOU

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